Conciseness of some words related to non-commutators

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Abstract

A group word $w = w(x_1, x_2 \dots, x_n)$ is an element of the free group F of rank n. If G is any group, we denote by G_w the set of all elements of G obtained by replacing $x_1, x_2, \dots, x_n$ in w by arbitrary elements $g_1, g_2, \dots, g_n$ of G, and by $w(G)$ the subgroup generated by G_w. We say that w is *concise* if $w(G)$ is finite for all G such that G_w is finite, and we say that w is *boundedly* concise if, whenever $|G_w| = m$, there exists a bound $\nu(w, m)$ such that $|w(G)| \leq \nu$. Well-known results in the subject are that all non-commutator words and all multilinear commutator words are concise, due to P. Hall, in an unpublished work, and Turner-Smith [3].

In [1] we provide good evidence for the following conjecture: if $u_1, u_2, \dots, u_n$ are non-commutator words in disjoint sets of variables, is $[u_1, u_2, \dots, u_n]$ concise? We will prove the validity of the conjecture for the case $n = 3$, as well as the case where the words u_i are the same, but in mutually disjoint sets of variables. This extends a Theorem of [2], where the result is proved for $n = 2$. Moreover, it is a well-know result, due to Turner-Smith [3], that if u is a concise word, then $[u, x]$ will also be, where x is a variable not appearing in u. The methods found by us give a similar result by proving that, when u is a multilinear commutator word and v is a non-commutator, then $[u, v]$ is concise. Some considerations about bounded conciseness in residually finite groups are also made. This is a joint work with P. Shumyatsky.

References

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